INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, First Semester, 2025-26
Introduction to Statistical Inference, Midterm Examination, 10th September, 2025

Answer all questions **Marks: 30**

Time: 2 hours

1. Solve the following questions (2×5) :

   (a) Let X_1, X_2, X_3, X_4 be independent random variables of sizes 4 with densities
   $$f_{X_i}(x_i|\alpha) = \begin{cases} \frac{3x^2}{\alpha^3} & \text{if } 0 < x < \alpha; \\ 0 & \text{otherwise,} \end{cases}$$
   where α is unknown parameter. If the observed values of the random sample are $2, 3, 6, 4$ then find the MLE of α.

   (b) Does bounded completeness imply completeness? If not, provide a counterexample.

   (c) Let X be a random variable from Normal population with mean μ and variance σ^2, where $-\infty < \mu < \infty,\ \sigma^2 > 0$ are unknown parameters. Find the Fisher Information matrix $I(\mu, \sigma^2)$.

   (d) Determine whether the following statement is correct. If not, give a counterexample:
   "If T is a sufficient statistic and V is a function of T, then V is also sufficient."

   (e) Let $X_1, \ldots, X_7$ be a sample from Normal population with mean 0 and variance $\theta > 0$. Let $K = \frac{\sum_{i=1}^{2} X_i^2}{\sum_{i=1}^{5} X_i^2}$, then show that the statistics K and $\sum_{i=1}^{5} X_i^2$ are independent.

2. Solve the following questions (2×10).

   (a) Let $X_1, \ldots, X_n$ be independent random variables of sizes $(n \geq 2)$ with densities
   $$f_{X_i}(x_i|\mu, \sigma) = \begin{cases} \frac{1}{\sigma} e^{-\frac{x-\mu}{\sigma}} & \text{if } x > \mu; \\ 0 & \text{otherwise,} \end{cases}$$
   where $\mu,\ \sigma > 0$ are unknown. (3+3+3+1)

   i. Show that $(X_{(1)}, \bar{X})$ is jointly sufficient for (μ, σ).

   ii. Find the Method of Moment estimator and Maximum Likelihood estimator of σ when μ is known. Are they same?

   (b) Let $X_1, \ldots, X_n \sim Poission(\lambda),\ \lambda > 0$. Let $T(X) = \sum_{i=1}^{n} X_i$ be a statistic. (3+2+1+4)

i. Show that $T(X) = \sum_{i=1}^{n} X_i$ is complete and sufficient statistic for λ.

ii. Consider $g(\lambda) = e^{-\lambda} = P(X_1 = 0),\ \lambda > 0$. Let

$$d(X_1) = \begin{cases} 1 & \text{if } X_1 = 0; \\ 0 & \text{otherwise.} \end{cases}$$

Show that $d(X_1)$ is an unbiased estimator of λ.

iii. Find the UMVUE of $g(\lambda)$.